

VALUE AT RISK IN THE SOUTH AFRICAN EQUITY MARKET: A VIEW FROM THE TAILS

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Abstract

Traditional parametric Value at Risk (VaR) estimates assume normality in financial returns data. However, it is well known that this distribution, while convenient and simple to implement, underestimates the kurtosis demonstrated in most financial returns. Huisman, Koedijk and Pownall (1998) replace the normal distribution with the Student's t distribution in modelling financial returns for the calculation of VaR. In this paper we extend their approach to the Monte Carlo simulation of VaR on both linear and non-linear instruments with application to the South African equity market. We show, via backtesting, that the t distribution produces superior results to the normal one.

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Introduction

The Value at Risk (VaR) of a portfolio of financial instruments at the confidence level x is given by the smallest number l such that the probability that the loss L exceeds l is no larger than $(1-x)$.¹ It is intended to calculate the maximum possible loss on the value of the portfolio over a specific time period with a certain level of confidence (McNeil et al., 2005) and answers the question 'How much can I lose with x per cent probability over a certain holding period?' (JP Morgan/Reuters, 1996). VaR is appealing in that it attempts to provide a single number summarising the market risk in a portfolio of assets and expresses this directly by assigning a monetary value to the potential losses in the portfolio (Hendricks, 1996; Hull, 2006). It has become a risk measure which is widely-used by financial institutions, both for internal risk management and regulatory reporting purposes and has also made its way into the Basel II capital-adequacy framework (Hendricks, 1996; Mc Neil et al., 2005, Van den Goorbergh, 1999).

Methodologies for calculating VaR are divided into parametric and non parametric approaches.

The difference between the two is in the way the distribution of expected returns is derived. In the former, returns are assumed to mimic a period in the past while the latter methodology assigns an actual probability distribution to the underlying risk factors.

The benefits of parametric versus non-parametric methods are discussed in the literature with mixed views expressed (see, for example, Brooks & Persaud, 2000). The main disadvantage of historical simulation, a non-parametric method, is the heavy reliance on past data since, under this method, the potential profits and losses on the portfolio under consideration are calculated by assuming that the returns that occurred in the past will be repeated in the future (Hull, 2006; Mc Neil et al., 2005). The VaR at a confidence level of x per cent is just the $(1-x)$ percentile of the potential profits and losses calculated using past returns. This implies that the VaR number used looks only at returns on one or two days whereas the parametric approach draws information from the entire distribution of returns. This is an advantage of parametric over non-parametric methodologies (Brooks & Persaud, 2000).

The common assumption in calculating VaR under the parametric approach is to assume that

financial returns follow a normal distribution. This is largely a function of simplicity and convenience since the normal distribution is specified with only two parameters and is easily applied to the calculation of portfolio profits and losses. In practice, financial returns exhibit kurtosis or 'fat tails' that are not captured by the normal assumption, notably, the number of returns in the tails is greater than would be expected from a normal distribution. In other words, financial returns show more extreme movements than would be predicted by the normal distribution (Resnick, 2007). Since VaR focuses on the tail of the distribution, or the extreme returns, attempting to quantify losses with a high degree of confidence (99 per cent for regulatory reporting), this insufficiency is particularly relevant here.² A number of alternative distributions such as Pareto and Sum Stable have been considered in the literature, but implementation has proved difficult (Huisman et al., 1998). Various other techniques for incorporating kurtosis have also been considered (see, for example, Hull & White, 1998), but these tend to lack the simplicity of assuming normality. The Student's t distribution is, however, much more successful than the normal distribution in capturing the fatness in the tails of financial returns' distributions and requires little more effort than the latter to implement.

This paper focuses on parametric methods. In particular, we extend the work of Huisman et al. (1998), who fit the Student's t distribution to financial return series, to include the calculation of VaR using Monte Carlo simulation as well as estimating the VaR on non-linear instruments. We apply the methodology to the South African equity market where we find that the Value at Risk estimated using the Student's t distribution is more conservative than the traditional parametric method that assumes a normal distribution of returns. We show this for a linear position in equity as well as for an investment in options (non-linear instruments). Since a single VaR number is insufficient proof that one model consistently outperforms another, we then apply the model to a period in history including the recent credit crunch and compare actual losses to those predicted by the VaR

models. This process is called backtesting. The results show that replacement of the normal distribution with the Student's t distribution in VaR calculations results in a consistently more accurate VaR calculation (both in number and size of exceptions). Thus using the t distribution reduces the capital charge for the financial institution. We also find that, due to the shape of the t distribution (fatter tails than the normal, but less pronounced in the region between the centre and the tails), the higher the confidence level for which we estimate VaR, the greater the performance of the t distribution relative to the normal one.

The paper is arranged as follows: in Section 2, we describe the dataset used. Since the most common assumption in parametric Value at Risk calculation methodologies is that the returns data are normally distributed, we begin with a brief empirical investigation of this assumption which is intended to show simply that the Student's t distribution provides a much better fit to the tails of the financial data than the normal distribution. Section 3 is a brief introduction to the t distribution, including a goodness-of-fit test, while Section 4 describes the methodology used in calculating the VaR of the equities and equity derivatives that we use as illustrative examples. In Section 5 we show the results obtained by applying both the normal and Student's t distributions to the South African equity market. Section 6 shows the results of backtesting the respective methods over the past year. Section 7 provides a conclusion and Section 8 some suggestions for further research.

2

Investigating the normality assumption

2.1 Data

A history of the daily closing level of the South African FTSE/JSE Top 40 index, which comprises forty South African equities weighted by market capitalisation, was obtained from Bloomberg for the ten year period beginning on 18 September 1998 and ending on 19 September 2008.

A history of daily closing prices for the following stocks was also obtained for the same period: Standard Bank (SBK), AngloGold Ashanti (ANG) and Pick 'n Pay (PIK). These are chosen as representative of the South African financial, mining and retail sectors respectively.

The daily log returns were then obtained by dividing each day's closing price by the previous day's close and taking the log of the return as follows:

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right), t = 2, 3, \dots, N$$

where r_t is the return³ for day t , $t = 2, 3, \dots, N$

S_t is the index level on day t , $t = 2, 3, \dots, N$

N is the number of daily closing prices in the time series

The summary statistics for the daily returns data are presented in the table below.

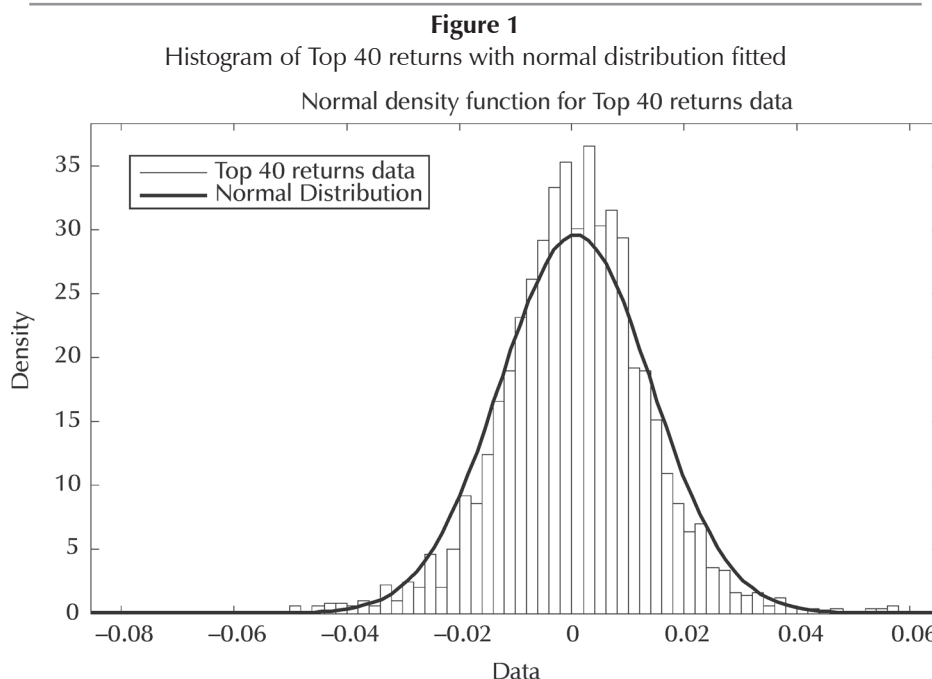
Table 1
Summary statistics for daily log returns time series

no. of returns	mean	median	maximum	minimum	standard deviation	skewness	excess kurtosis
2,500	0.07%	0.09%	6.39%	-8.39%	0.0135	-0.0883	2.5319

2.2 Viewing tails graphically

Fitting a normal distribution to a histogram⁴ of the entire history of the Top 40 daily returns data

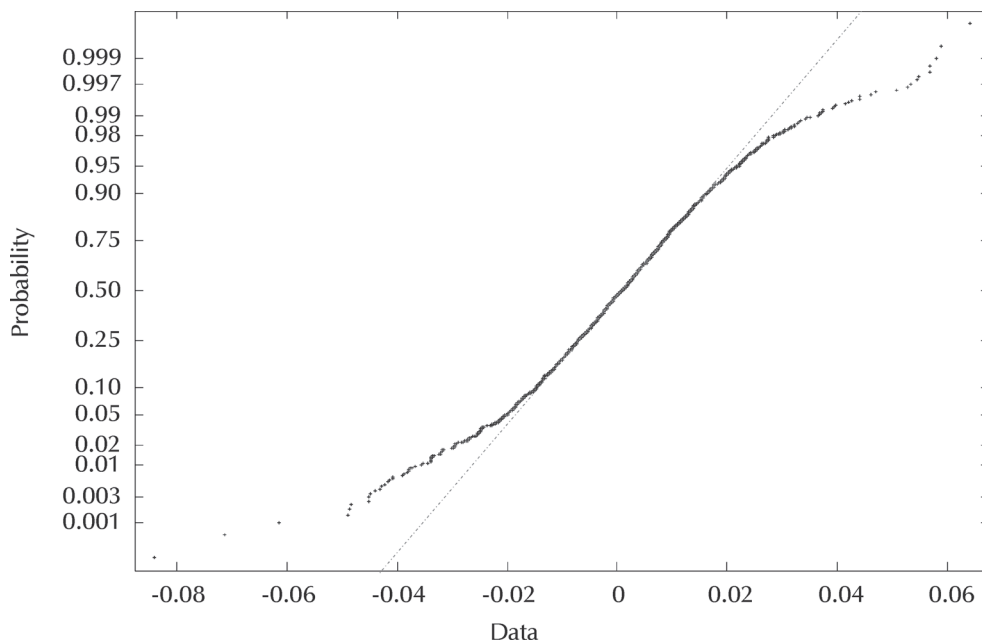
reveals the poor fit in the tails of the distribution as shown in Figure 1 below.



This is perhaps even more apparent in the QQ plot in Figure 2 below. Here, normally distributed data fall on the straight line. It is clear that the differences between the daily returns data in our sample and the normal

distribution are most apparent in the tails which are crucial in estimating Value at Risk. The reason for the emphasis on the tails is that, in calculating VaR, it is extreme losses with which we are concerned.

Figure 2
QQ plot for Top 40 returns data



3 Introducing the Student's t distribution

The Student's t distribution is characterised by the degrees of freedom (DoF) parameter. The DoF or tail index measures the speed at which the tail approaches zero, that is to say, the speed at which the probabilities of extreme events approach zero. Thus, the fatter the tail, in other words, the greater the kurtosis exhibited by the data, the slower the speed with which the tail approaches zero and the lower the index. When the degrees of freedom approach infinity, the t distribution approaches normality (Evans et al., 1993 and Huisman et al., 1998). The rest of the characteristics of the normal distribution,

such as symmetry, are thus echoed by Student's t . This means that, in applying the latter in calculating VaR, we are correcting for only one of the potentially many flaws encountered by using the normal distribution. However, in estimating VaR, the lack of kurtosis in the normal distribution is arguably the most critical assumption.

The graphs above (figures 1 and 2) are repeated below, but this time with the t distribution fit illustrated next to the normal one. These show clearly that the t distribution is more appropriate in accounting for the tails in the Top 40 returns data than the normal distribution. This is even more apparent in Figure 5 below where we zoom in on the right tail of the distribution.

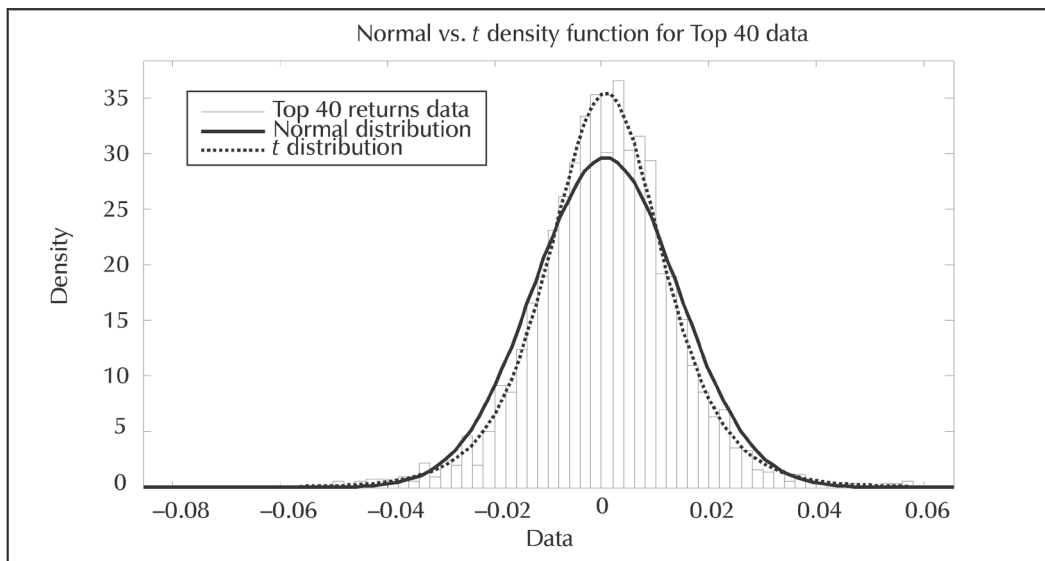
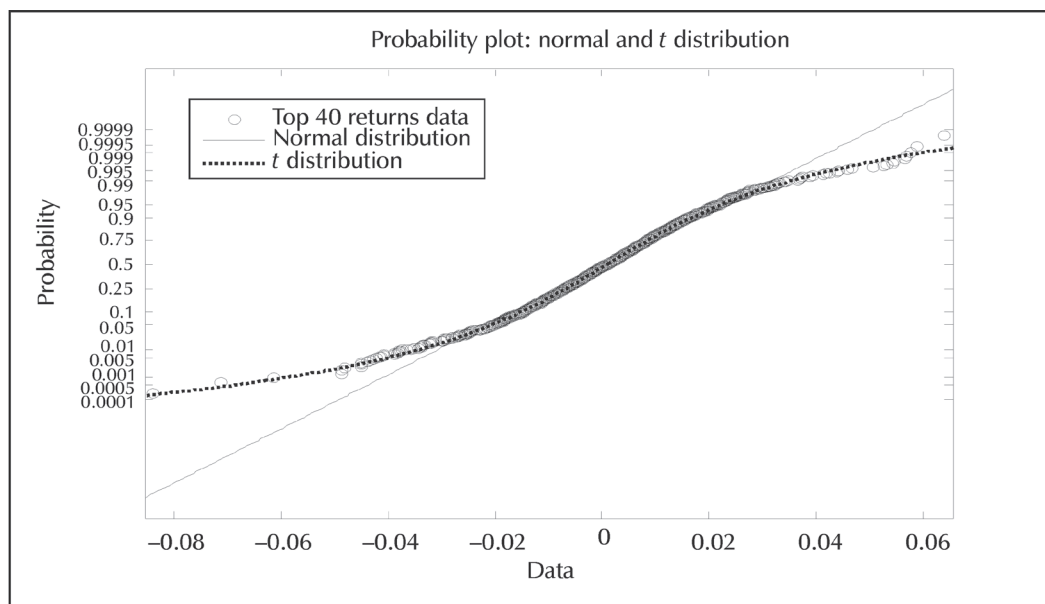
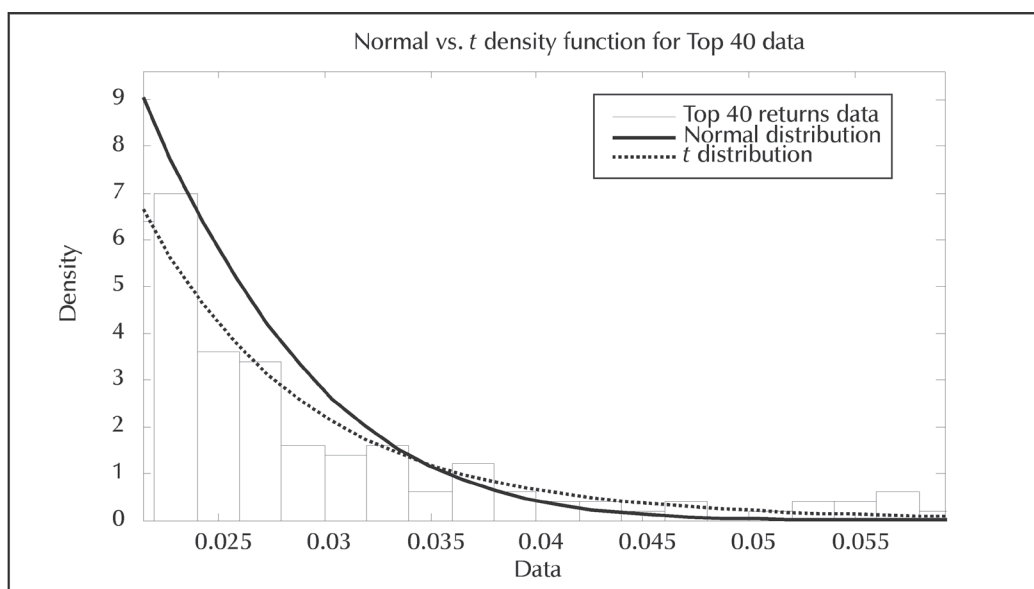
Figure 3Histogram of Top 40 returns with normal and t distributions fitted**Figure 4**Probability plots with normal and t distributions fitted

Figure 5Right tail of the histogram of top 40 returns with normal and t distributions fitted

A Chi-square goodness-of-fit test is used to assess the hypothesis that a set of data comes from a specified distribution against the alternative that it does not⁵. For the Top 40 returns series discussed in Section 2.1 of this paper, we conducted two Chi-square goodness-of-fit tests, one for the hypothesis that the data follow a normal distribution and one for the hypothesis that they come from a Student's t distribution. In the first test, the null hypothesis of normality was rejected while in the latter, the null hypothesis of a Student's t distribution could not be rejected at a 95 per cent level of confidence⁶. While the results serve only to cement our case, the superiority of the t distribution can clearly be seen in the tails in the above graphs and that is the portion of the distribution with which we are most concerned in the calculation of VaR.

4

Methodology

4.1 Monte Carlo simulation

The calculation of VaR via Monte Carlo simulation has the advantage of being extremely

flexible in the diversity of instruments for which it caters. While the method can be time consuming, there are a number of techniques available for speeding up computation time. We use Monte Carlo simulation for all VaR calculations in this paper. We focus on Daily Value at Risk (DVaR) which is the VaR for a one day holding period.

The method involves generating many scenarios for the underlying risk factors. The value of the underlying stock price or index level is calculated according to the formula:

$$S_{t+1}(i) = S_t e^{\sigma \varepsilon(i)} \quad (1)$$

where S_t is the level of the index or equity spot price on day t (the most recent day for which we have information)

$S_{t+1}(i)$ is the simulated value of the spot price in scenario i on day $t+1$, the day for which we are calculating VaR

σ is the daily volatility of the stock price/index level returns⁷

$\varepsilon(i)$ is a standard normal random variable when the underlying distribution of returns is assumed to be normal and $\varepsilon(i)$ is sampled from the Student's t distribution with the appro-

priate degrees of freedom when the underlying returns' distribution is assumed to be the Student's t .⁸

The portfolio for which we are calculating VaR is then revalued in each of the scenarios for which we have simulated spot prices and the profit and loss calculated in each scenario. The $(1-\alpha)^{\text{th}}$ percentile of the generated portfolio values is then the VaR at an α^{th} level of confidence (Hull, 2006). In other words, we expect the loss on the portfolio to breach this value only $(1-\alpha)$ per cent of the time.

When the scenarios are generated from a t distribution, we follow the terminology of Huisman et al. (1998) and call the Value at Risk number generated VaR-X. When the usual assumption of normality is employed, we call the VaR number VaR-N. This will avoid confusion later in the paper.

4.2 Parameterising the Student's t distribution

The number of degrees of freedom of the Student's t distribution is estimated using the Hill estimator. This methodology was first published by Hill in 1975 (Hill, 1975) and is applied by Huisman et al. (1998).⁹

The Hill Estimator is given as:

$$\gamma(k) = \frac{1}{k} \sum_{j=1}^k \ln x_{n-j+1} - \ln x_{n-k} \quad (2)$$

where x_i is the i^{th} increasing order statistic based on absolute values of observations and k is the number of tail observations (Hill, 1975). Thus the estimator is a function of the number of tail observations specified.

A modified version of the Hill Estimator for small samples can also be utilised (see Huisman et al., 1998).

The tail index or degrees of freedom of the Student's t distribution is then the inverse of the Hill Estimator (Huisman et al., 1998).

5 Results

5.1 VaR-X versus VaR-N – linear position

For the three representative stocks as well as the index in our dataset, we estimate the DoF of the Student's t distribution using the full history obtained and assuming that an observation three standard deviations from the mean constitutes a tail event.¹⁰

For a linear position of 10,000 ALSI contracts (assuming a contract size of 10 South African Rand (ZAR) per point) and long positions of 1 000 000 shares in each of the three single stocks, we obtain the results for VaR-N and VaR-X on 18 September 2008 for a one day holding period in Table 2 below. These values are calculated using Monte Carlo Simulation as described in Section 4.

Table 2

Value at Risk results at various confidence levels on 18 September 2008

Stock/Index	DoF	99.90%		99.00%		98.00%	
		VaR-X	VaR-N	VaR-X	VaR-N	VaR-X	VaR-N
Top 40	4.55	1 706 938 338	1 190 262 805	1 008 131 638	898 450 153	838 180 369	795 580 457
SBK (Financial)	4.48	1 062 697 634	636 240 778	549 960 482	487 752 645	459 470 908	431 284 066
ANG (Mining)	4.50	2 205 868 946	1 624 950 623	1 460 071 092	1 241 297 647	1 175 380 269	1 093 752 604
PIK (Retail)	4.11	336 384 584	208 652 636	192 927 541	158 913 177	152 584 146	141 465 956

Table 2 Continued

Stock/Index	DoF	97.00%		96.00%		95.00%	
		VaR-X	VaR-N	VaR-X	VaR-N	VaR-X	VaR-N
Top 40	4.55	741 182 819	733 876 294	661 770 921	687 517 783	613 107 078	639 429 172
SBK (Financial)	4.48	395 922 526	389 444 571	359 865 898	366 040 272	329 509 102	343 932 671
ANG (Mining)	4.50	1 030 836 892	993 280 860	920 187 477	937 273 273	860 614 520	886 599 257
PIK (Retail)	4.11	131 467 830	128 784 048	116 925 201	118 415 013	108 229 753	112 293 615

It is clear from the above table that, at high confidence levels, VaR-X produces a more conservative VaR estimate than VaR-N. As we move further out of the tails, in other words,

as we lower the confidence level of the VaR estimate, VaR-N becomes more appropriate than VaR-X. This is illustrated in Figures 6 to 9 below.

Figure 6
VaR-X and VaR-N for Top 40

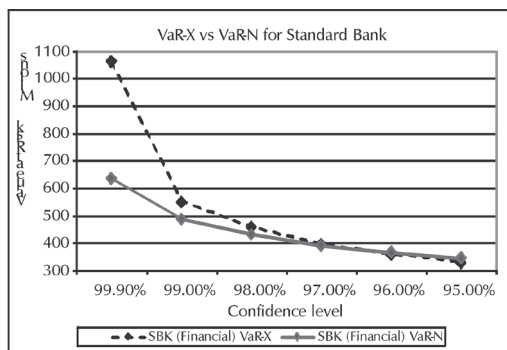


Figure 7
VaR-X and VaR-N for SBK

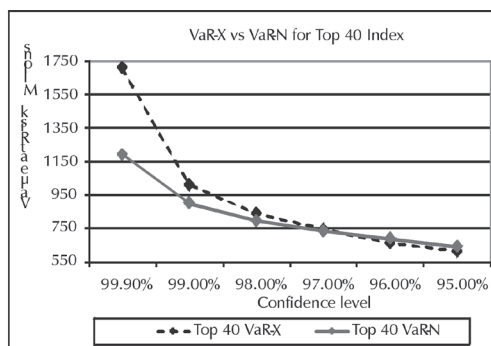


Figure 8
VaR-X and VaR-N for ANG

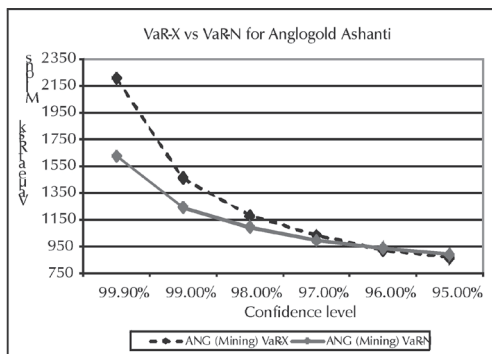
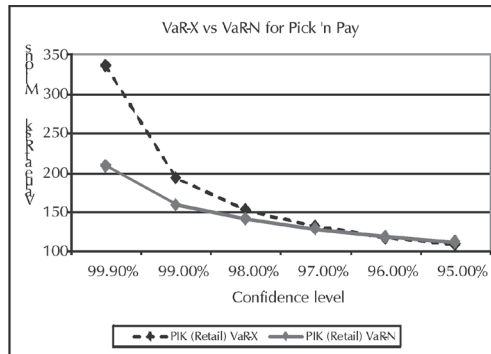


Figure 9
VaR-X and VaR-N for PIK

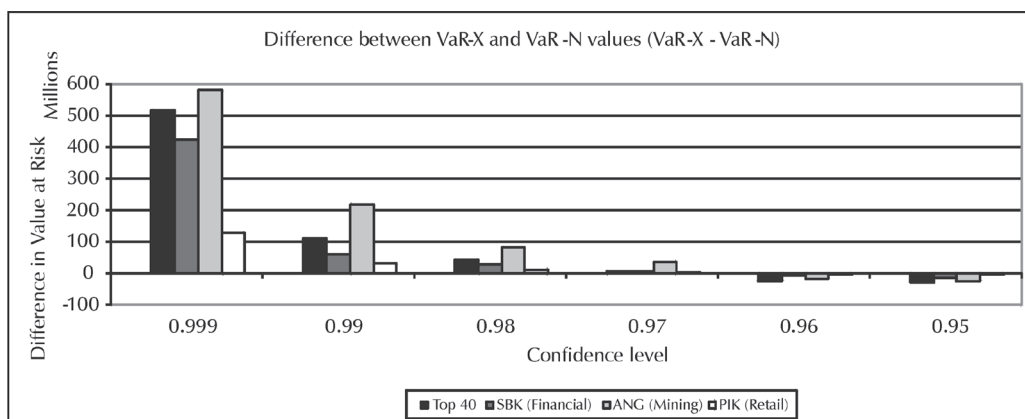


The above graphs also show that the magnitude of the difference between VaR-X and VaR-N increases the deeper into the tail we move. The actual differences between VaR-X and VaR-N are illustrated in Figure 10 below.

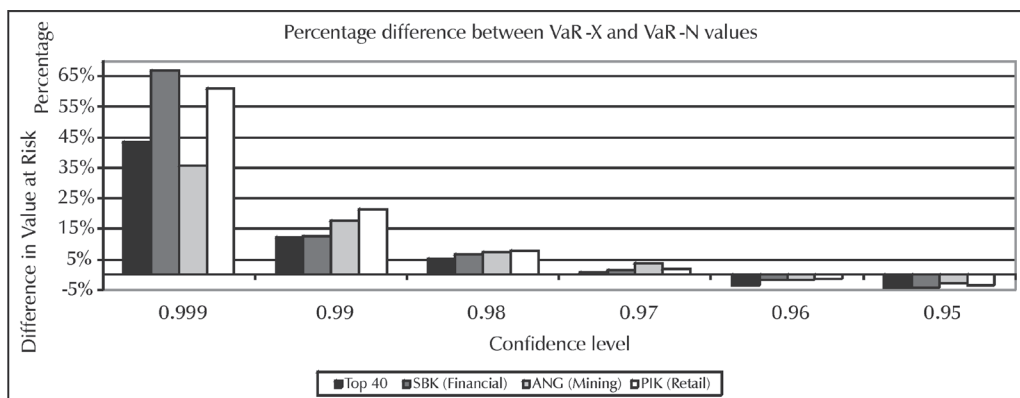
In Figure 11 these differences are scaled by the VaR-N value which shows that the relative VaR difference is not merely attributable to the Value at Risk being larger for a higher confidence level.

Figure 10

Actual differences between VaR-X and VaR-N for various confidence levels

**Figure 11**

Percentage differences between VaR-X and VaR-N for various confidence levels



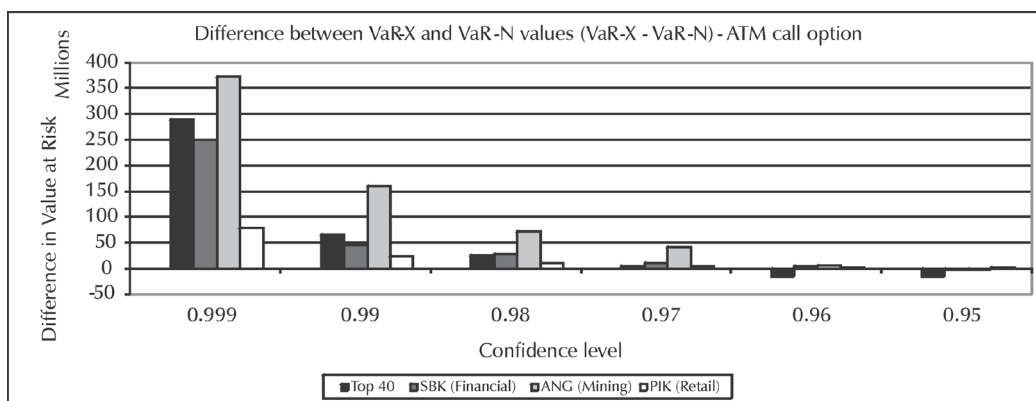
As mentioned above, since regulatory VaR is calculated at a 99 per cent significance level, it is clear that a distribution such as Student's t with greater kurtosis than the normal distribution is important in calculating VaR. These differences in risk numbers could be magnified many times in the large trading books held by financial institutions.

5.2 Extension to non-linearity

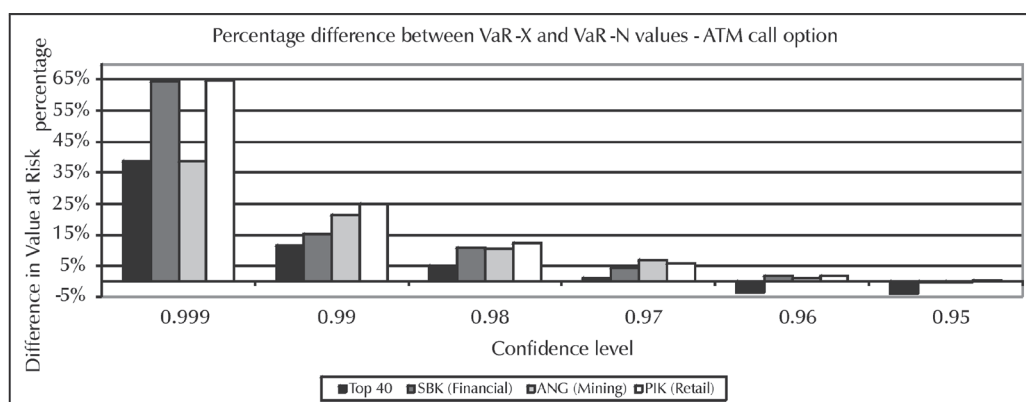
Having considered a linear position in the index and stocks, we now look at the more complex case of a call option, with the index and stocks as the underlying reference entities¹¹. The Value at Risk is then calculated for 1 000 000 ATM call options on the index and each of the stocks. The results are illustrated in Figures 11 and 12 below. The percentage differences and patterns in the VaR numbers produced are similar to those for the linear position above. However, for the single stocks, the VaR-X estimate becomes more conservative than the VaR-N value from less deep in the tails.

Figure 12

Difference between ATM call option VaR-N and VaR-X values

**Figure 13**

Percentage difference between ATM call option VaR-N and VaR-X values



5.3 Varying the degrees of freedom

Since the parameterisation of the Student's t distribution is dependent on the number of tail observations, k , in the dataset, which is partly a subjective choice, we calculate the VaR on 18 September 2008 for various choices of k (based on the number of standard deviations from the

mean we term tail observations as well as the size of the dataset).

The table below shows the DoF parameter that gave the lowest VaR number. Where 'normal' appears in the table, the VaR-N value was more conservative than any of the VaR-X values.

Table 3

DoF value producing most conservative VaR estimate for various confidence levels

Confidence level	99.9%	99.0%	98.0%	97.0%	96.0%	95.0%
Top 40	3.43	4.00	4.00	4.63	Normal	Normal
SBK	3.64	3.64	4.48	5.29	Normal	Normal
ANG	4.10	4.50	5.46	4.50	10.48	Normal
PIK	2.91	4.11	4.11	4.11	Normal	Normal

Table 3 shows that the DoF parameter that gives us the most conservative VaR tends to decrease with an increase in the confidence level with the exception of the 97–98th percentile of Anglogold Ashanti.¹² This suggests that the parameterisation of the t distribution might be enhanced when based on the confidence level for which we wish to estimate VaR.

The table also shows that VaR-X is most appropriate deep in the tails of the distribution while VaR-N is more appropriate from about a 96 per cent confidence level and lower. The reason for this is illustrated in Figure 3, where it is obvious that the density function of the Student's t distribution gives higher probabilities in the centre of the distribution and deep in the tails while the normal probability is higher between the two.

Using VaR-X instead of VaR-N might therefore not be an improvement for an institution calculating VaR at a 95 per cent level of confidence. However, many institutions calculate VaR for 98 or 99 per cent confidence levels and indeed, regulatory VaR is calculated at a 99 per cent confidence level. For

such institutions and those holding regulatory capital, VaR-X provides a more accurate VaR estimate.

6

Backtesting the model

Since VaR is intended to calculate the maximum loss which is not exceeded with a given level of confidence (McNeil et al., 2005), the proof of the new methodology lies in measuring how the VaR-X that would have been estimated in the past would have predicted actual losses incurred versus the results for VaR-N.

The Basel Committee requires backtesting to be performed over the last 250 trading days where 99 per cent Daily Value at Risk (DVaR)¹³ is compared to realised daily returns over the period. The number of exceptions, particularly days on which actual losses are higher than those predicted by VaR, determine the amount of capital required to be held by the bank. This is shown in Table 4 below (Hurlin & Tokpavi, 2006).

Table 4
Number of exceptions with corresponding scaling factor

Zone	Exceptions	Scaling factor
Green	0-4	3
Yellow	5	3.4
	6	3.5
	7	3.7
	8	3.8
	9	3.9
Red	10 or more	4

The backtesting is here conducted assuming a constant investment of ZAR1m. Using the most recent 250 days of Top 40 data, the position size of a ZAR1m investment is calculated each day and the actual profit/loss for the day is calculated using the difference in the value of the investment between that day and the next due to the change in the level of the Top 40 index. The Value at Risk that would have been calculated for each of the 250 days for the ZAR1m position

is then compared to actual losses. When the size of the loss exceeds the VaR number, an exception is recorded.

We compare backtesting results for both VaR-N and VaR-X for various positions in the Top 40, including both long and short linear positions and a long call and put position. The DoF of the Student's t distribution is estimated out of sample and is updated quarterly. The backtesting results are shown in Table 5 below.

Table 5

Backtesting results for 99 per cent DVaR for Top40 positions over most recent 250 trading days in our data sample.

Position	No. of exceptions		Percentage of exceptions		Zone		Sum of exception sizes (ZAR)		Average exception size (ZAR)*	
	VaR -N	VaR-X	VaR-N	VaR-X	VaR-N	VaR-X	VaR-N	VaR-X	VaR-N	VaR-X
Linear - long	5	2	2.00%	0.80%	Yellow	Green	-38,539	-17,762	-13,057	-13,057
Linear - short	6	3	2.40%	1.20%	Yellow	Green	-77,546	-56,709	-22,652	-18,903
Non linear - call	5	2	2.00%	0.80%	Yellow	Green	-168,256	-81,714	-59,425	-40,857
Non linear - put	6	3	2.40%	1.20%	Yellow	Green	-423,237	-297,397	-99,132	-120,360

*The average exception size refers to the average for days on which both models showed an exception. Using the average of the VaR-N exceptions could possibly produce a lower average value than for VaR-X due to small exceptions on VaR-N on days where VaR-X showed none.

With reference to Table 5, we measure the number of exceptions as well as the percentage of our backtesting sample this includes. In addition, we look at the actual sizes of the exceptions. Since DVaR estimates losses, the amount by which actual losses exceed the VaR number is relevant. The column showing the 'Sum of Exception sizes' in Table 5 is the cumulative amount by which VaR-N and VaR-

X underestimate losses for the 250 trading days in our testing sample. The 'Average Exception Size' shows the average size of exceptions on days where both models underestimated losses. Clearly VaR-X outperforms VaR-N in every possible way. Graphs showing the actual profit or loss of the various positions against VaR-N and VaR-X estimates are shown below.

Figure 14

Backtesting results for long linear ZAR1m rolling investment

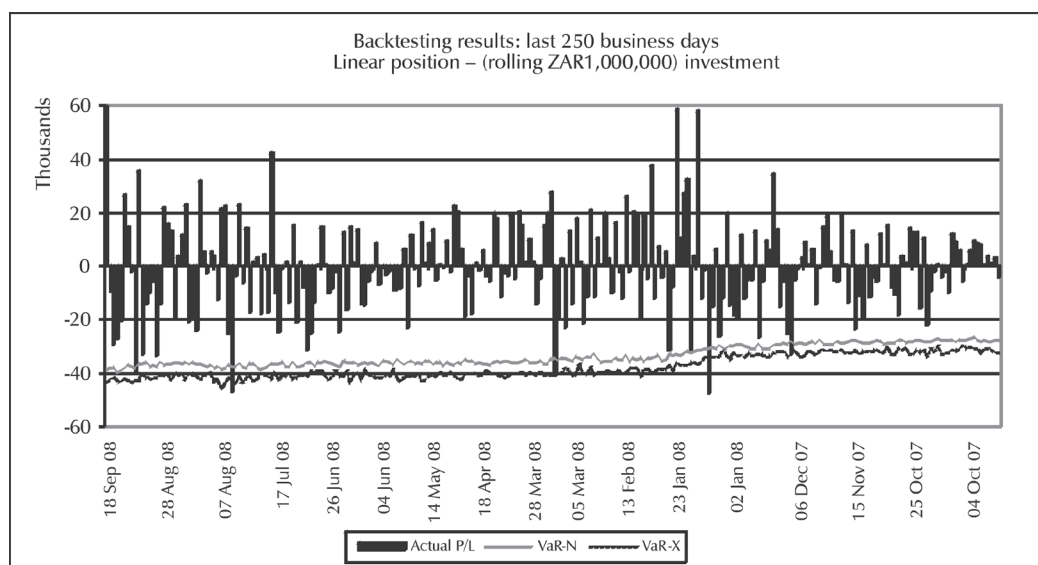
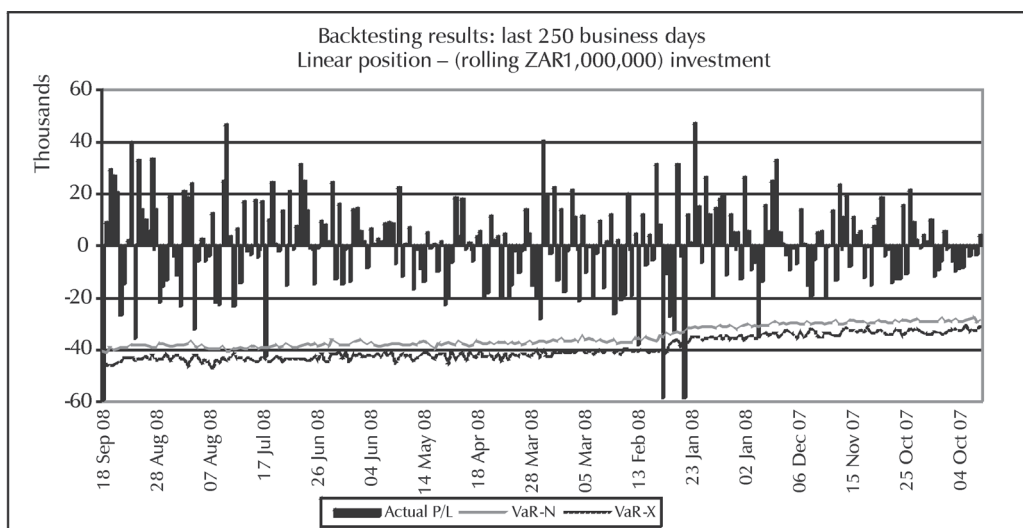


Figure 15

Backtesting results for short linear ZAR1m rolling investment

**Figure 16**

Backtesting results for rolling ZAR1m call option investment

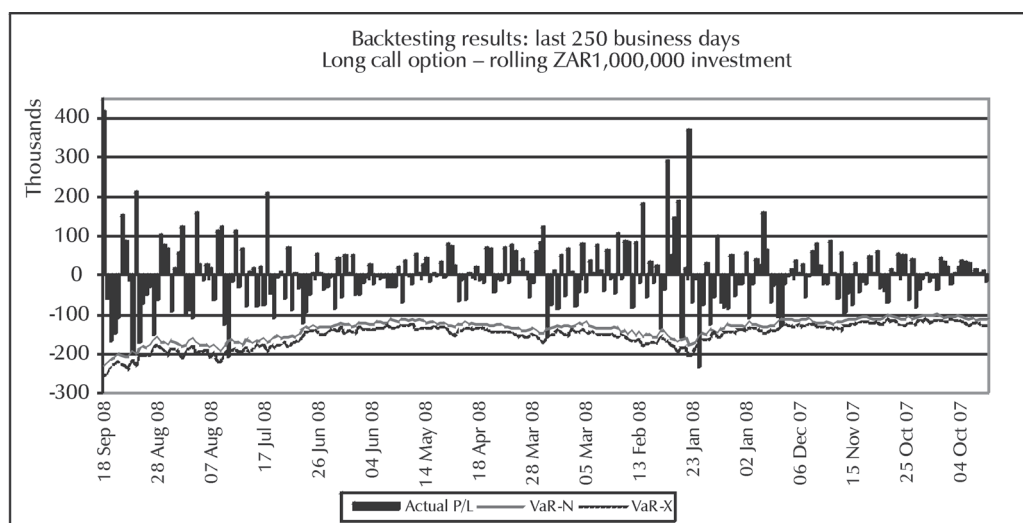
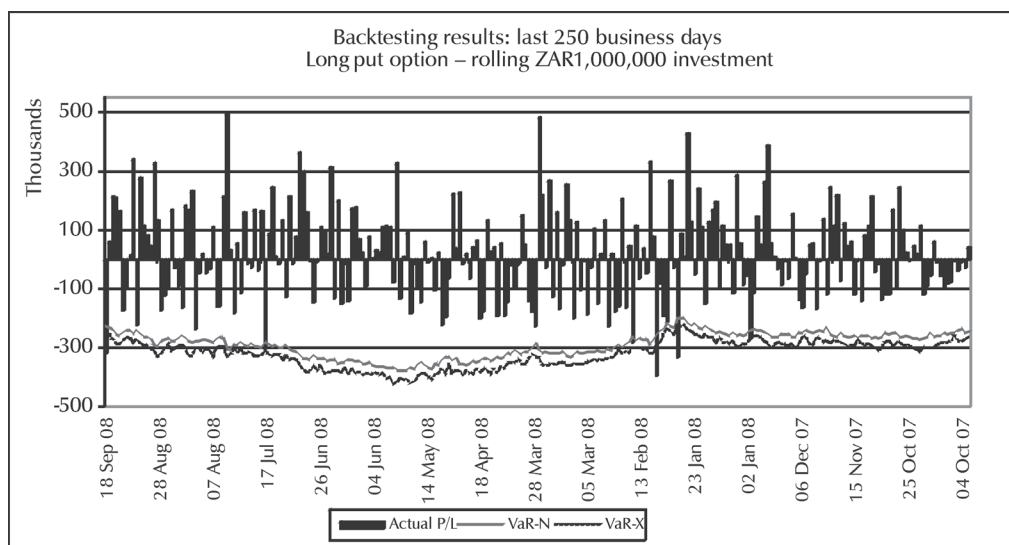


Figure 17
Backtesting results for rolling ZAR1m put option investment



7 Conclusion

It is commonly accepted that financial returns' distributions exhibit kurtosis or heavier tails than the normal distribution. We demonstrated that this phenomenon extends to the South African equity market.

We then extended the methodology of Huisman et al. to the calculation of VaR via Monte Carlo simulation for both linear and non linear instruments. The results presented clearly show that the VaR-X methodology outperforms VaR-N in estimating daily losses on positions in the Top 40 index and three representative South African stocks as well as in options on these. Not only are the number of exceptions in backtesting lower for VaR-X than for VaR-N, but the amount by which the value at risk underestimates losses is also lower, thus minimising unexpected losses. The results show that an institution using VaR-X for regulatory reporting would have the lowest possible multiplier in determining capital to be held for regulatory purposes while the institution using VaR-N would be required to hold more.

With the recent volatility observed in financial markets, the need for a model that anticipates tail events in a superior manner to the normal

distribution has become clear. The Student's t distribution provides one such model when used for estimating financial returns. The results presented in this paper were achieved by updating the degrees of freedom only quarterly. The advantage of this distribution is that it is easily parameterised and achieves a VaR number with little more effort than when the normal distribution is used.

8 Directions for further research

While academics and practitioners in modern finance agree that the normal distribution is inadequate to model the risks posed by moves in financial asset prices, there is not a single answer to the question of which distributional assumptions to use.

Much work has been done in the literature to investigate distributional assumptions such as power laws (see, for example, Malevergne, Pisarenko & Sornette, 2005) and generalised extreme value distributions (see, for example, De Haan & Ferreira, 2006). All these methods, including the t distribution described in this paper, have in common that they improve on the assumption that returns are normally distributed.

In our analysis we have demonstrated that the t distribution fits easily to data and performs well on the basis of back-testing as far as VaR calculation is concerned. The t distribution is therefore a good candidate to replace the general assumption of normality. There is undoubtedly the same case to be made for power law or generalised extreme value distributions, albeit that these have different implementation issues in practice. This is clearly an area of further research interest.

An additional alternative that is easily implemented, at least for the linear case, is the Cornish-Fisher-Expansion¹⁴ (see for example Jaschke, 2001). This comes with its own concerns, which are not discussed here. Table 6 below shows the VaR-N and VaR-X of a linear position in the Top 40 as well as the Cornish-Fisher-Expansion results for the position. The numbers support further research into the latter methodology although there does seem to be overestimation of VaR in the extreme tails¹⁵. Backtesting and extension to the non-linear case may well be worth investigating.

Table 6

Comparison of VaR-N and VaR-X values with the Cornish-Fisher-Expansion for a linear 10 000 contract position in the Top 40

Confidence level	Method		
	VaR-N	VaR-X	Cornish-Fisher
99.9%	1 190 262 805	1 706 938 338	1 796 392 929
99.0%	898 450 153	1 008 131 638	1 410 048 711
98.0%	795 580 457	838 180 369	1 024 567 405
97.0%	733 876 294	741 182 819	823 654 354
96.0%	687 517 783	661 770 921	692 781 282
95.0%	639 429 172	613 107 078	598 226 355

The application of the t distribution could also be extended to the case of more complex exotic derivatives particularly those where tail events play a large role such as in credit derivatives.

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Endnotes

- Formally, VaR_x

$$= \inf\{l \in R: P(L > l \leq 1 - x)\}$$

$$= \inf\{l \in R: FL(l) \geq x\}$$
(McNeil et al., 2005).
- The kurtosis inherent in financial returns is not the only way in which the distribution of returns differs

- from a normal distribution. Another example of the imperfections of the normal distribution when applied to financial data is the skewness present in many financial time series. This factor is not addressed in this paper since the Student's t distribution, presented here as an alternative to the normal distribution, is also symmetrical.
- In the sequel the terms returns and log returns will be used interchangeably following standard practice in finance and risk management (McNeil et al., 2005).
- Matlab's 'dfittool' was used to fit the distribution to the data as well as to plot the graphs in Figures 1, 2, 3 and 4.
- The test statistic is $c = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}$ where O_i is the observed frequency for bin i and E_i is the expected frequency for bin i . The decision is to reject the null hypothesis that the data follows a specific distribution if $c \geq \chi^2_{k-p-1\alpha}$ where k is the number of bins into which we divide the data, p is the number of parameters in the distribution against which we're testing. For more information

on this and other goodness-of-fit tests, see for example Steyn et al. (1994).

- 6 The value of the test statistic in the first test (for normality) was 161 (due mostly to the underestimation of tail events) and that of the second test (for the Student's t test) was 5.
- 7 In this paper, we took the volatility to be the standard deviation of the most recent 250 days' returns. More sophisticated volatility estimation methods could be used such as volatility updating (JP Morgan/Reuters, 1996), but the aim of the paper is to compare results generated from the t distribution to those generated from the normal distribution.
- 8 Note that since the standard statistical tables provide t random numbers for a distribution with variance equal to $\text{dof}/(\text{dof}-2)$, the numbers sampled have to be scaled by dividing by $\sqrt{\text{dof}/(\text{dof}-2)}$ (Evans et al., 1993, Huisman et al., 1998 and Mc Neil et al., 2005).
- 9 For alternative methods to the Hill estimator of specifying the degrees of freedom of the t Distribution see for example Resnick, 2007 and Van den Goorbergh, 1999.
- 10 Since there is no set rule for determining whether or not an observation constitutes a tail event, this is not necessarily the decision that has to be applied. It was decided to use the 3 standard deviation rule in this paper as it consistently produced good results and it was necessary to make a decision on what observations to use in parameterising the t distribution so that results could be compared with one another and a consistent rule was applied ensuring that comparisons were not biased.
- 11 The details of the call options are given in the table below.

	Top 40	SBK	ANG	PIK
Spot	23,259 points	R 89	R 196	R 31
Strike	23,259 points	R 89	R 196	R 31
r	12.00%	12.00%	12.00%	12.00%
vol	20.15%	34.30%	39.86%	31.97%
tenor (yrs)	1	1	1	1
div yield	3.60%	2.00%	2.00%	2.00%

- 12 The difference between the 97 per cent VaR-X value with a DoF parameter of 4.5 and 5.46 for this stock is only R11 per point and both values are more conservative than the VaR-N estimate.
- 13 The daily VaR or DVaR is the VaR for a holding period of one day.
- 14 Cornish-Fisher VaR is calculated as follows:

$$Z_{cf} = q_p + \frac{(q_p^2 - 1)S}{6} + \frac{(q_p^3 - 3q_p)K}{24} - \frac{(2q_p^5 - 5q_p)S^2}{36}$$

$$VaR = -\bar{R} - \sigma Z_{cf}$$
 where q_p is the p 'th percentile of the standard normal distribution, \bar{R} is the mean return and S is the skewness and K the kurtosis of the returns series (see VaR 101 reference).
- 15 While we don't wish to underestimate risk, overestimation would result in a waste of capital.

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