Econometric estimation of Armington elasticities for selected agricultural products in South Africa

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Abstract
Price transmission behaviour is used to model the impacts of different trade regimes; if this behaviour is not modelled correctly, the trade impacts can be either under- or overestimated. Due to the lack of elasticities of substitution pertaining to selected imported and domestically produced agricultural products in South Africa, Armington elasticities, using quarterly data from 1995-2006 and three different models, based on the time series properties of the data, are estimated in this paper. Considering the long-run elasticity results, soyabean (whether broken or not) and meat of bovine animals (frozen) are the most sensitive import products, followed by maize, meat of bovine animals (fresh or chilled), sunflower seeds, and wheat and meslin. Regarding the short-run elasticity, soyabean is the most sensitive import product, followed by meat of bovine animals (fresh or chilled); meat of swine (fresh, chilled or frozen) is the least sensitive import product.

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1 Introduction
The economic evaluation of, for example, trade liberalisation requires complex models that can take different forms and are based on economic theory. Of particular importance in computable partial and general equilibrium models used to model the impacts of different trade regimes are the behavioural functions that govern the interactions between different variables. For instance, in these models, changes in trade regimes and tariffs alter the domestic price of imported goods relative to that of the domestically produced goods, and such changes in relative prices affect the share of the demand which is supplied by imports (Tourinho, Kume & Pedroso 2003). If such behaviour is not modelled correctly, the impact of different trade policy regimes can be either under- or overestimated, which in turn could result in the implementation of inefficient trade policies. One therefore needs estimates of the elasticity of substitution between goods differentiated by their place of origin to properly measure how, for example, a more liberal trade policy regime will affect a domestic industry. This elasticity is formally known as the Armington elasticity.

Moreover, according to Gallaway, McDaniel and Rivera (2003), when economic models are used to evaluate changes in trade policy, converting policy changes to price effects is very important. Trade policy models use these price shifts to determine how the policy under
review will affect output, employment, trade flows, economic welfare and other variables of interest. Trade model parameters are commonly expressed in the form of elasticities. It represents the percentage change of one variable in response to a one per cent change in another variable, all other things being equal. Elasticities are rooted in micro-economic theory and reflect consumers’ and firms’ sensitivity to changes in relative prices and income (Hertel, Ianchovichina & McDonald, 1997).

Estimates of Armington elasticities are not available for agricultural products in the majority of countries, including South Africa, in spite of the importance of including Armington elasticities for evaluating the impact of trade policies. One frequently encounters studies in this area where researchers use Armington elasticity estimates for other countries as proxies to substitute for the required Armington elasticities of their own countries and, in many cases, this completely disregards the important differences between foreign countries’ and their home country’s structure of production and consumption.

A review of the literature revealed that Armington elasticities are non-existent for the major agricultural products in South Africa. The aim of this research is to estimate Armington elasticities for the following products (as specified in the Harmonised System of codes), namely: meat of bovine animals (fresh or chilled); meat of bovine animals (frozen); meat of swine (fresh, chilled or frozen); maize (corn); wheat and meslin; soyabeans (whether broken or not), and sunflower seeds (whether broken or not). The estimated Armington elasticities can be used in future trade related research, e.g., in studies that include South Africa and where partial or general equilibrium models are applied to better represent the substitution effects (imports vs. domestically produced) of the mentioned products.

These products mentioned above were selected based on their use of natural resources, their relative importance in terms of their contributions to the gross value of agricultural production, and their tradability.

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**Figure 1:**
The contribution of selected agricultural products to the gross value of agricultural production

![Graph showing percentage contribution of selected agricultural products from 1996/97 to 2007/08](source: DAFF, 2009)
In South Africa, approximately 68 per cent of total land is utilised for grazing purposes (DAFF, 2009), i.e. utilisation by the extensive livestock sub-sector such as beef. In addition the chosen products contributed, on average, 31 per cent to the gross value of agricultural production between 1996/97 to 2007/08 (see Figure 1). Significantly, white maize and wheat (bread) are regarded as being vitally important staple foods, while sunflower seed and oilseeds are perceived as being important sources of edible oils and animal feeds.

According to Oyewumi (2006), South Africa’s commitment to trade liberalisation has resulted in strong growth in import demand. He provides ample evidence of the growth in imports for the products mentioned above, except for maize. In addition, recent trade policy questions, e.g., tariffs policies, centre around these products; evidence of this is provided when looking at the number of submissions to the International Trade Administration Commission (ITAC) for consideration to amend current tariff regimes.

The remainder of this paper is organised as follows: The next section briefly discusses approaches used to estimate the Armington elasticity, as well as the general form of the Armington model; Section 3 presents a review of selected studies using the Armington approach; in Section 4 the data used are discussed, while in Section 5 the econometric methodology used in this paper is derived; finally, Section 6 presents the results of the analysis, while concluding remarks are provided in the final section.

2 Approaches used to estimate Armington elasticities and the general form of the Armington model

The literature describes two common approaches to empirically estimate Armington elasticities, namely validation and econometric estimation. The econometric approach has been criticised based on the following: firstly, given the large number of parameters to be estimated, long run time series data for numerous variables is required to provide sufficient degrees of freedom for estimation; secondly, the economy is likely to have undergone structural changes over time, which may or may not be appropriately reflected in the estimation procedure; and thirdly, the values of estimates usually seem to vary widely, depending on the time series data used, the functional form used and the methodology adopted (Arndt, Robinson & Tarp, 2002).

As an alternative to the econometric approach, some researchers using computable general equilibrium (CGE) models employ a simple ‘validation’ procedure. The results from this approach can provide a basis for revising estimates of some important parameters, i.e., recalibrating the model in a kind of informal Bayesian estimation procedure. Unlike econometric approaches, this approach makes very limited use of the historical record and provides no statistical basis for judging the robustness of estimated parameters (Arndt, Robinson & Tarp, 2002).

Combining the two methods described above, Arndt, Robinson and Tarp (2002) adopted an entropy-based approach to estimate elasticity parameters for CGE models. Compared with other approaches, this new approach has the advantage of endogenously determining the ‘general equilibrium’ values of the model’s behavioural parameters (including substitution elasticities), which are also consistent with historical observations. There are, however, also limitations to this approach. The results are dependent on an entropy ratio statistic, which is known to have weak predictive power. As the results are dependent on selected historical targets, as with other back-casting type approaches, this approach also requires a relatively large amount of historical data from external sources, which opens the possibility of data inconsistency (Zhang, 2006).

The econometric method of estimation is used in this study due to the availability of sufficient time series data. The rest of this section presents the general form of the Armington model. The Armington model is a prudent model that shares some elements of both neoclassical and advanced trade models. The main theoretical background of this model is that goods imported by a country from the rest of the world are considered imperfect substitutes for goods made

The introduction of Armington substitution in the demand for commodities is a departure from the assumption of perfect substitution that underlies traditional trade models (Lloyd & Zhang, 2006). This departure fundamentally changes the properties of a trade model and the well-known theoretical results that are based on variants of the Heckscher-Ohlin model (Lloyd & Zhang, 2006).

The Armington assumption of product differentiation and imperfect substitution makes existing trade statistics immediately usable for global trade models. The Armington structure also overcomes the problem that arises in a Heckscher–Ohlin-type model with more goods than factors, whereby countries tend to specialise in only a few of the goods.

In the first stage, a representative consumer allocates total expenditures to different product categories. In the second stage, a representative consumer allocates expenditure within both the domestic and imported goods groups, taking relative prices as given (Gallaway, McDaniel & Rivera, 2003). Thus, following Armington (1969) and much of the ensuing literature, it was assumed that consumer utility for goods in an industry is separable from consumption of other products, and postulates a simple CES sub-utility function to model demand for home and imported goods in that industry:

\[
U(M,D) = \alpha \left[ \beta M^{(\sigma - 1)/\sigma} + (1 - \beta)D^{(\sigma - 1)/\sigma} \right]^{\sigma/(\sigma - 1)}
\]

Where:

- \( U \) = sub-utility over the home and foreign goods
- \( M \) = quantity of the imported goods
- \( D \) = quantity of domestic goods
- \( \sigma \) = constant elasticity of substitution between domestic and imported goods
- \( \alpha \) and \( \beta \) = parameters in the demand function

A complete mathematical derivation of the model is available in Appendix A.

### 3 Review of selected studies using the Armington approach

A review of the literature revealed that extensive work has been done on the estimation of Armington elasticities for industrial products, but less has been done on agricultural products. From the international literature reviewed, studies by Stern, Francis and Schumacher (1976), Shiells, Stern and Deardorff (1986), Reinert and Roland-Holst (1992), and Shiells and Reinert (1993) provided valuable trade substitution elasticities. These studies, however, did not carefully consider the time series
properties of the data and they did not explicitly consider the long-run aspect of the Armington elasticity that is applicable to applied partial and general equilibrium modelling.

Kapuscinski and Warr (1999), Gallaway, McDaniel and Rivera (2003) and Tourinho, Kume and Pedroso (2003) employed techniques that took care of the time series properties of the data and they estimated long-run Armington elasticities. The study by Gallaway, McDaniel and Rivera (2003) is unique when compared to the other two studies in that it employed techniques that distinguish between the short- and long-run elasticities.

Similar studies for South Africa mostly ignored the stationary nature of the time series data; if the data series employed is non-stationary, generated estimates may be misleading and unreliable for use in applied modelling work. Van Der Merwe and Van Heerden (1997), Naude (1999) and Burrows (1999) applied specifications that do not allow extraction of both short- and long-run elasticities. Gibson (2003), however, overcame all of these problems by applying the specification of Gallaway, McDaniel and Rivera (2003) to estimate short- and long-run Armington industrial elasticities. Gibson’s study offers the latest and most appropriate set of Armington elasticities for the South African industrial sector. This specification, as used by Gibson (2003), uniquely considered the long-run aspect of Armington elasticities as being applicable to applied partial and general equilibrium modelling. This approach also takes into consideration the issue of stationarity, thereby avoiding spurious regression analysis.

4 Data used

In order to estimate the Armington elasticities using the models that are derived in the next section of this paper, four different data series were used. These were real imports, domestic sales of domestically produced goods and the prices of the two groups of goods (i.e., the local and the international price). Data used to construct the appropriate data series were sourced from the Trade and Industrial Policy Strategies (TIPS), South Africa Standardized Industry Indicator Data Base, and the Department of Agriculture. Data on import and export quantities and values were sourced from TIPS, while domestic production data in real and current terms were sourced from the Department of Agriculture. The data are quarterly in nature, from the first quarter of 1995 to the third quarter of 2006.

The series’ ‘domestic sales of domestically produced goods’ is constructed by subtracting ‘exports’ from total production both in real and current terms. The former is used as the domestic sales series of domestically produced goods. The ratio of constant to current domestically consumed output of domestically produced goods generates a suitable domestic sales price index for each of the products.

The Laspeyres Index was used to calculate the real import series using the year 2000 as the base year, and is as follows: Let \( m_x \) represent the monthly import quantity of the 4 digit HS product \( x \) in the time period \( t \), and \( v \) represent the 2000 average monthly unit value of product \( p \). The real import series is calculated as:

\[
M_t = \sum_x v_x \times m_x
\]

The price series were calculated using the formula:

\[
Pm_t = \left( \sum_i CV_i \right) / M_t
\]

The final step to calculate the real import series used in the estimation was to normalise the import quantity series so that the average quarterly 2000 value of \( M_t \) equalled 1. This series was then multiplied by the value of imports for the 2000 fourth quarter to obtain a series of the same magnitude as the value of imports for the HS category in 2000. Real quarterly exports were constructed using the same procedure as imports.

5 Econometric methodology

5.1 Specification

Gallaway, McDaniel and Rivera (2003) specified three different models that can be estimated based on the time series properties of the data series employed per product. These models are
adaptations from the traditional Armington model described in Section 2. These three models were also used by Gibson (2003) for the South African industrial sector. The three models are: the geometric lag model, single equation error correction model and the ordinary least squares model.

Seasonality is an important characteristic of agricultural products that must be taken into consideration. In order to reflect the characteristics of seasonality, quarterly dummies were included in the specification of the Armington equations for the agricultural products under consideration. Where the time variable was found to be important, dummies were also included.

First, for products having stationary log level data, a parsimonious geometric lag model was estimated because it can be used to easily extract both short-run and long-run elasticity estimates. Therefore, equation (3) was specified as:

\[ y_t = \alpha + \beta_0 x_t + \lambda y_{t-1} + b_1 D_2 + b_2 D_3 + b_3 D_4 + v_t \]  

Where \( y \) and \( x \) are the goods and price ratios, respectively, and are defined as such in equation (3). \( v \) represents an independently and identically distributed (iid) error term while \( D_2, D_3 \) and \( D_4 \) are dummy variables representing the second, third and fourth quarters of the year. Long-run elasticity estimates can be estimated as \( \beta_0 / (1 - \lambda) \) if \( 0 < \lambda < 1 \); otherwise, the reported elasticities are \( \beta_0 \). Lags were considered in equation (4) because the process of adjusting the quantity of import and domestic goods in response to exogenous price changes may take some time to complete. Therefore, lags were used to capture the dynamic relationship between quantity and price.

Secondly, when the series is both I(1) and co-integrated, a single equation error correction model of the following form was estimated to extract the long-run elasticity estimates:

\[ \Delta y_t = \alpha + \beta_0 \Delta x_t + \beta_2 \Delta y_{t-1} + \beta_3 x_{t-1} + b_1 D_2 + b_2 D_3 + b_3 D_4 + v_t \]  

Where \( \Delta y_t = y_t - y_{t-1} \) and \( v \) represents an (iid) error term, \( D_2, D_3 \) and \( D_4 \) are dummy variables representing the second, third and fourth quarters of the year. This model allows the short- and long-run responses of demand with respect to price to be determined. Short-run elasticity estimates are \( \beta_0 \) and long-run elasticity is \( (\beta_0 / \beta_1) \). Equation (5) is an unrestricted version of ECM obtained by re-parameterising equation (4) (see Stewart, 2005).

Finally, when only one of the series was stationary, the variables were first differenced for stationarity and an ordinary least square regression was estimated:

\[ \Delta y_t = \alpha + \beta_0 \Delta x_t + b_1 D_2 + b_2 D_3 + b_3 D_4 + v_t \]  

Where \( \beta_0 \) is the short-run Armington elasticity and \( D_2, D_3 \) and \( D_4 \) are dummy variables representing the second, third and fourth quarter of the year. This equation does not yield long-run values; this is because there is no long-run relationship between the goods and the price ratio series.

5.2 Diagnostic tests

For each estimated equation, a series of diagnostic tests was performed. The Augmented Dickey Fuller (ADF) and Phillips and Perron (PP) test were performed for each of the series to determine whether they have unit root or not. Where there was a contradiction between the ADF result and the PP result, the results from the PP test is preferred (Obben, 1998). The justification is that the ADF test is based on the hypothesis that the series is generated by an autoregressive (AR) process, while the PP test is based on the more general autoregressive integrated moving-average (ARIMA) process (Tang, 2003). Time series that were found to be I(1) were tested for co-integration to reveal the existence of a long-run relationship. Recursive estimates of log level data were conducted to detect any outliers that may distort the value of coefficient estimates. Outliers were controlled in this estimation by using dummy variables for the year(s) concerned. The Lagrange multiplier test of residuals serial correlation was also performed
on the estimates and, where autocorrelation was detected, the Cochrane–Orcutt iteration method was used to correct for it. All other important validation parameters are reported in the results. Results of the unit root test and co-integration tests are available in Appendix B.

6 Results

The estimated short- and long-run Armington elasticities for the selected agricultural products are presented in Table 1. Also included in the table are the estimated equations and important summary regression statistics. The price series in equation (2) is inverted, thus the elasticity estimates are positive. All the products considered in this study have a significant Armington elasticity at 10 per cent level of significance. All the products except meat of swine (fresh, chilled or frozen) have short- and long-run elasticities (note: meat of swine is the only product that required the use of equation (6)). Equation (5) was estimated for meat of bovine animals (frozen) while equation (4) was estimated for all other products. Short-run Armington elasticities range from 0.79 to 3.47 and the long-run elasticities range from 1.91 to 4.50. Long-run elasticities are on average larger than the short run elasticities.

Table 1:
Short- and long-run Armington elasticities estimates for agricultural products in South Africa

<table>
<thead>
<tr>
<th>HIS</th>
<th>Description</th>
<th>Eq</th>
<th>Armington elasticity</th>
<th>R²</th>
<th>Adj-R²</th>
<th>DW stat</th>
<th>Quarterly dummies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Short-run</td>
<td>Long-run</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0201</td>
<td>Meat of bovine animals, fresh or chilled</td>
<td>4</td>
<td>2.54 (0.000)</td>
<td>2.76 (0.000)</td>
<td>0.63</td>
<td>0.58</td>
<td>1.41</td>
</tr>
<tr>
<td>0202</td>
<td>Meat of bovine animals, frozen</td>
<td>5</td>
<td>1.21 (0.033)</td>
<td>3.21 (0.000)</td>
<td>0.36</td>
<td>0.31</td>
<td>1.94</td>
</tr>
<tr>
<td>0203</td>
<td>Meat of swine, fresh, chilled or frozen</td>
<td>6</td>
<td>0.79 (0.032)</td>
<td>0.43</td>
<td>0.39</td>
<td>2.03</td>
<td>–</td>
</tr>
<tr>
<td>1005</td>
<td>Maize (corn)</td>
<td>4</td>
<td>2.03 (0.000)</td>
<td>2.75 (0.000)</td>
<td>0.78</td>
<td>0.76</td>
<td>1.87</td>
</tr>
<tr>
<td>1001</td>
<td>Wheat and meslin</td>
<td>4</td>
<td>1.28 (0.000)</td>
<td>1.91 (0.000)</td>
<td>0.82</td>
<td>0.80</td>
<td>1.79</td>
</tr>
<tr>
<td>1201</td>
<td>Soyabeans, whether broken or not</td>
<td>4</td>
<td>3.47 (0.000)</td>
<td>4.50 (0.000)</td>
<td>0.83</td>
<td>0.81</td>
<td>1.93</td>
</tr>
<tr>
<td>1206</td>
<td>Sunflower seeds, whether broken or not</td>
<td>4</td>
<td>1.65 (0.000)</td>
<td>2.03 (0.000)</td>
<td>0.82</td>
<td>0.80</td>
<td>1.95</td>
</tr>
</tbody>
</table>

Note: Only significant estimates are recorded and their P-values are in brackets. Value of $\lambda$ does not exceed 1 in all the cases equation (4) was estimated.

Also reported in Table 1 are the dummy variables representing seasonality. Dummy variables for livestock products are found to be statistically not significant except for those of swine meat (fresh, chilled or frozen) during quarter four. However, the dummy variables for the grain products are statistically significant. This suggests that seasonality is an important factor in determining import demand for grain products.
Following Tourinho, Kume and Pedroso (2003), the estimated Armington elasticity can be classified using a very high, high, average, low, null and wrong sign. The elasticity and classification is presented in Table 2. Based on this classification, it can be concluded that the long-run Armington elasticities for bovine animal meat (frozen) and soyabeans (whether broken or not) are very high. Meat of bovine animals (fresh or chilled), maize (corn), sunflower seeds (broken or not broken) and wheat and meslin have high Armington elasticities. The relatively high value of the Armington elasticity means that imports and domestic supplies of the commodity are considered by households to be virtually identical, and the degree of substitution is close. When the elasticity is much lower than infinity, it can be concluded that the imported and domestic goods are imperfect substitutes.

### Table 2:
Classification of Armington elasticities (Long-run elasticities)

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>Relative sensitivity</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>Larger than 3</td>
<td>Very high</td>
<td>0202 (Meat of bovine animals, frozen), 1201 (Soyabeans, whether broken or not)</td>
</tr>
<tr>
<td>Between 1.5 and 3</td>
<td>High</td>
<td>0201 (Meat of bovine animals, fresh or chilled), 1005 (Maize: corn), 1206 (Sunflower seeds, broken or not broken), 1001 (Wheat and meslin).</td>
</tr>
<tr>
<td>Between 0.5 and 1.5</td>
<td>Average</td>
<td>None</td>
</tr>
<tr>
<td>Less than 0.5</td>
<td>Low</td>
<td>None</td>
</tr>
<tr>
<td>Non-significant</td>
<td>Null</td>
<td>None</td>
</tr>
<tr>
<td>Negative</td>
<td>Wrong sign</td>
<td>None</td>
</tr>
</tbody>
</table>

Considering the long-run elasticity results, soyabeans (whether broken or not) and meat of bovine animals (frozen) are the most sensitive import products, followed by maize, meat of bovine animals (fresh or chilled), sunflower seeds and wheat and meslin. Regarding the short-run elasticity, soyabeans are the most sensitive import product, followed by meat of bovine animals (fresh or chilled); meat of swine (fresh, chilled or frozen) is the least sensitive import product. From a policy point of view, this means that products that exhibit a high level of sensitivity could be highly vulnerable to changes in, for example, tariff regimes. This in turn could affect the ability of local producers to sustain local production. This is typically information that should be considered by the International Trade Administration Commission when considering changes to tariffs of agricultural products.

The estimated Armington elasticities for meat of bovine animals (fresh or chilled) are 2.54 and 2.76 for the short- and long-run, respectively. This means that, all things being equal, if the domestic price of meat of bovine animals (fresh or chilled) increases by 1%, then the quantity of this product imported by South Africa from its trading partners will increase by 2.54 per cent in the short-run and by 2.76 per cent in the long-run. This product is considered to be a very sensitive product based on its Armington elasticity value. The other Armington elasticities can be interpreted in the same way.

### 7 Conclusion

In applied models that are used to, for example, examine the impact of changes in trade policies, it is important that cognisance is taken of the fact that changes in relative prices affect the fraction of the demand which is supplied by imports. If price transmission behaviour is not modelled correctly, trade impacts can be either under- or
overestimated. One therefore needs estimates of the elasticity of substitution between goods differentiated by their place of origin. In this study, short- and long-run Armington elasticities were estimated for seven agricultural products using quarterly data from 1995 to 2006.

Different econometric models were used for the different agricultural products included in the study. The most appropriate model was determined according to the statistical characteristics of the time series.

The long-run elasticities range from 1.91 to 4.50, and the short-run Armington elasticities range from 0.79 to 3.47. These values suggest that imported and domestic agricultural products are far from perfect substitutes. The long-run elasticity estimates show that soyabeans (whether broken or not) and meat of bovine animals (frozen) are the most sensitive import products, followed by maize, meat of bovine animals (fresh or chilled), sunflower seeds, and wheat and meslin. Regarding the short-run elasticity, soyabeans are the most sensitive import product, followed by meat of bovine animals (fresh or chilled); meat of swine (fresh, chilled or frozen) is the least sensitive import product. Finally, the results show that seasonality is an important factor in determining import demand for grain products.

The value of research of this nature is that the estimated Armington elasticities will allow researchers to more precisely evaluate the economic impacts of trade liberalisation, changes in tariffs and other trade policies in partial and general equilibrium models that include South African agriculture.

It is, however, recommended that Armington elasticities for other agricultural products with a relatively high trade percentage relative to domestic production also be estimated using a similar methodological approach. In addition, such a study should take note that this study has not considered stocks as part of the aggregate availability of grains due to data limitations, but future studies should attempt to include stocks as it could potentially influence the ‘willingness’ to import. Also, shifts in trade regimes, i.e., moving from being an importer to an exporter and vice versa, should be addressed in more detail. The majority of the studies applying the Armington model to agricultural trade deal mainly with bulk commodities. Trade modelling in processed food products has received little attention. This set of products is important because of their differentiated nature and the growing importance of these products in world trade. It is therefore important that further research is also done in this area.

8 References


Appendix A

Following Armington (1969) and much of the ensuing literature, it is assumed that consumer utility for goods in a country is separable from consumption of other products, and a simple CES sub-utility function is postulated to model demand for domestically produced and imported goods in that country:

\[ U(M, D) = \alpha \left[ \beta M^{(\sigma-1)/\sigma} + (1 - \beta) D^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \]  

(A.1)

Where:

\[ U = \text{sub-utility over the domestic and foreign goods} \]
\[ M = \text{quantity of imported goods} \]
\[ D = \text{quantity of domestic goods} \]
\[ \sigma = \text{constant elasticity of substitution between domestic and imported goods} \]
\[ \alpha \text{ and } \beta = \text{are calibrated parameters in the demand function} \]

Assuming that ‘\( \rho \)’ equals price, prices of imports and domestically produced goods are denoted as \( \rho_M \) and \( \rho_D \). In order to maximise expenditure, prices are made equal to the marginal utility derived from purchasing the associated products so that

\[ \delta U/\delta M = \rho_M \text{ and } \delta U/\delta D = \rho_D. \]

Thus, differentiating equation (A.1) with respect to M and D yields the following:

\[ \delta U/\delta M = \sigma/(\sigma - 1) \left[ \beta M^{(\sigma-1)/\sigma} + (1 - \beta) D^{(\sigma-1)/\sigma} \right]^{1/(\sigma - 1)} (\sigma - 1/\sigma) \beta M^{(-1)/\sigma} \]

\[ = \beta M^{(-1)/\sigma} \left[ \beta M^{(\sigma-1)/\sigma} + (1 - \beta) D^{(\sigma-1)/\sigma} \right]^{1/(\sigma - 1)}, \]  

(A.2)

Also,

\[ \delta U/\delta D = \sigma/(\sigma - 1) \left[ \beta M^{(\sigma-1)/\sigma} + (1 - \beta) D^{(\sigma-1)/\sigma} \right]^{1/(\sigma - 1)} (\sigma - 1/\sigma) (1 - \beta) D^{(-1)/\sigma} \]

\[ = (1 - \beta) D^{(-1)/\sigma} \left[ \beta M^{(\sigma-1)/\sigma} + (1 - \beta) D^{(\sigma-1)/\sigma} \right]^{1/(\sigma - 1)}, \]

Given that \( \delta U/\delta M = \rho_M \) and \( \delta U/\delta D = \rho_D \), then \( \rho_D/\rho_M \) can be rewritten as:

\[ \rho_D/\rho_M = (\delta U/\delta D)/(\delta U/\delta M) \]

\[ = (1 - \beta) D^{(-1)/\sigma} \left[ \beta M^{(\sigma-1)/\sigma} + (1 - \beta) D^{(\sigma-1)/\sigma} \right]^{1/(\sigma - 1)} \left[ \beta M^{(-1)/\sigma} \left[ \beta M^{(\sigma-1)/\sigma} + (1 - \beta) D^{(\sigma-1)/\sigma} \right]^{1/(\sigma - 1)} \right] \]

\[ = \left[ (1 - \beta) D^{(-1)/\sigma} \right]/\left[ \beta M^{(-1)/\sigma} \left[ \beta M^{(\sigma-1)/\sigma} + (1 - \beta) D^{(\sigma-1)/\sigma} \right]^{1/(\sigma - 1)} \right] \]

\[ = \left[ (1 - \beta)/\beta \right] \left[ M^{1/\sigma} / D^{1/\sigma} \right] \]  

(A.3)

Rearranging A.3 gives

\[ M/D = \left( \rho_D/\rho_M \right)^{\sigma}/(1 - \beta)/\beta \]

\[ = \left[ \left( \rho_D/\rho_M \right)^{\sigma} \right]/\left[ \beta (1 - \beta) \right]^{\sigma} \]

\[ = \left[ \beta (1 - \beta) \left( \rho_D/\rho_M \right) \right]^{\sigma} \]  

(A.4)

The first-order condition can be rewritten as:

\[ y = a_0 + a_1 x \]  

(A.5)

Where \( y = \ln(M/D), \) \( a_0 = \sigma \ln \left[ \beta/(1 - \beta) \right], \) \( a_1 \) is the elasticity of substitution between imports and domestic sales, and \( x \) represents \( \ln \left( \rho_D/\rho_M \right). \)
Appendix B

Table B.1:
Test statistics for unit roots in variables

<table>
<thead>
<tr>
<th>HS Code</th>
<th>Series</th>
<th>Levels</th>
<th>95% Critical value</th>
<th>1st difference</th>
<th>95% Critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0201</td>
<td>LNY</td>
<td>-5.1174*</td>
<td>-2.9266</td>
<td>-11.7588*</td>
<td>-2.9281</td>
</tr>
<tr>
<td></td>
<td>LNX</td>
<td>-4.1391*</td>
<td>-1.9481</td>
<td>-12.3593*</td>
<td>-1.9483</td>
</tr>
<tr>
<td>0202</td>
<td>LNY</td>
<td>-2.2565</td>
<td>-2.9266</td>
<td>-8.8085*</td>
<td>-2.9281</td>
</tr>
<tr>
<td></td>
<td>LNX</td>
<td>-1.0114</td>
<td>-1.9481</td>
<td>-7.6778*</td>
<td>-1.9483</td>
</tr>
<tr>
<td>0203</td>
<td>LNY</td>
<td>-5.4675*</td>
<td>-3.5107</td>
<td>-18.3934*</td>
<td>-3.5131</td>
</tr>
<tr>
<td></td>
<td>LNX</td>
<td>-1.7343</td>
<td>-1.9481</td>
<td>-7.4465*</td>
<td>-2.9320</td>
</tr>
<tr>
<td>1005</td>
<td>LNY</td>
<td>-4.0717*</td>
<td>-1.9481</td>
<td>-14.6295*</td>
<td>-1.9483</td>
</tr>
<tr>
<td></td>
<td>LNX</td>
<td>-4.9414*</td>
<td>-2.9266</td>
<td>-20.1065*</td>
<td>-2.9281</td>
</tr>
<tr>
<td>1001</td>
<td>LNY</td>
<td>-6.1617*</td>
<td>-1.9481</td>
<td>-19.3722*</td>
<td>-1.9483</td>
</tr>
<tr>
<td></td>
<td>LNX</td>
<td>-5.8372*</td>
<td>-1.9481</td>
<td>-24.2008*</td>
<td>-1.9483</td>
</tr>
<tr>
<td>1201</td>
<td>LNY</td>
<td>-6.2302*</td>
<td>-3.5107</td>
<td>-18.3063*</td>
<td>-3.5131</td>
</tr>
<tr>
<td></td>
<td>LNX</td>
<td>-5.2259*</td>
<td>-3.5107</td>
<td>-18.5288*</td>
<td>-3.5131</td>
</tr>
<tr>
<td>1206</td>
<td>LNY</td>
<td>-5.7263*</td>
<td>-2.9266</td>
<td>-16.0944*</td>
<td>-2.9281</td>
</tr>
<tr>
<td></td>
<td>LNX</td>
<td>-6.4885*</td>
<td>-2.9266</td>
<td>-15.6008*</td>
<td>-2.9281</td>
</tr>
</tbody>
</table>

Asterisk indicates statistical significance at 5 %.

Note: Y represents quantity variable and X represents price variable.

Table B.2:
Results of co-integration test

<table>
<thead>
<tr>
<th>Product</th>
<th>Test</th>
<th>Test statistics</th>
<th>95 % Critical value</th>
<th>90 % Critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0202</td>
<td>Max Eigen value test r = 0</td>
<td>20.2249*</td>
<td>14.8800</td>
<td>12.9800</td>
</tr>
<tr>
<td></td>
<td>Trace r = 0</td>
<td>23.8193*</td>
<td>17.8600</td>
<td>15.7500</td>
</tr>
<tr>
<td></td>
<td>Max eigen value and trace test: r = 1</td>
<td>3.5944</td>
<td>8.0700</td>
<td>6.5000</td>
</tr>
</tbody>
</table>

Asterisk indicates statistical significance at 5 %.